

The Quantum Mechanical Model of the Atom

Richard Parsons, (RichardP)
Sharon Bewick, Ph.D. (SharonB)

Say Thanks to the Authors

Click <http://www.ck12.org/saythanks>

(No sign in required)

To access a customizable version of this book, as well as other interactive content, visit www.ck12.org

CK-12 Foundation is a non-profit organization with a mission to reduce the cost of textbook materials for the K-12 market both in the U.S. and worldwide. Using an open-content, web-based collaborative model termed the **FlexBook®**, CK-12 intends to pioneer the generation and distribution of high-quality educational content that will serve both as core text as well as provide an adaptive environment for learning, powered through the **FlexBook Platform®**.

Copyright © 2013 CK-12 Foundation, www.ck12.org

The names “CK-12” and “CK12” and associated logos and the terms “**FlexBook®**” and “**FlexBook Platform®**” (collectively “CK-12 Marks”) are trademarks and service marks of CK-12 Foundation and are protected by federal, state, and international laws.

Any form of reproduction of this book in any format or medium, in whole or in sections must include the referral attribution link <http://www.ck12.org/saythanks> (placed in a visible location) in addition to the following terms.

Except as otherwise noted, all CK-12 Content (including CK-12 Curriculum Material) is made available to Users in accordance with the Creative Commons Attribution/Non-Commercial/Share Alike 3.0 Unported (CC BY-NC-SA) License (<http://creativecommons.org/licenses/by-nc-sa/3.0/>), as amended and updated by Creative Commons from time to time (the “CC License”), which is incorporated herein by this reference.

Complete terms can be found at <http://www.ck12.org/terms>.

Printed: May 15, 2013

flexbook
next generation textbooks



AUTHORS

Richard Parsons, (RichardP)
Sharon Bewick, Ph.D. (SharonB)

EDITOR

Shonna Robinson, (ShonnaR)

CHAPTER

1

The Quantum Mechanical Model of the Atom

CHAPTER OUTLINE

- 1.1 The Dual Nature of Light
 - 1.2 Characteristics of Matter
 - 1.3 Quantum Numbers, Orbitals, and Probability Patterns
-

1.1 The Dual Nature of Light

Lesson Objectives

The student will:

- name the model that replaced the Bohr model of the atom.
- explain the concept of wave-particle duality.
- solve problems involving the relationship between the frequency and the energy of a photon.

Vocabulary

- black body radiation
- diffraction
- interference
- photoelectric effect
- photon
- quantum
- quantum mechanics
- quantum theory
- wave-particle duality

Introduction

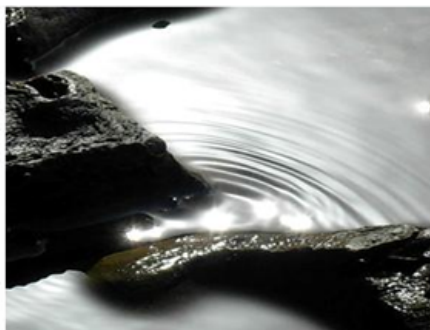
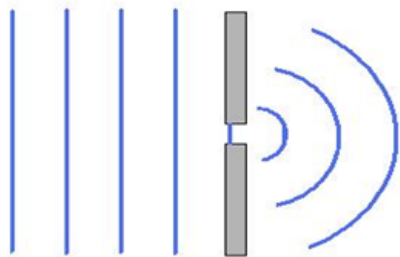
Further development in our understanding of the behavior of electrons in an atom's electron cloud required some major changes in our ideas about both matter and energy.

The branch of physics that deals with the motions of objects under the influence of forces is called mechanics. Classical mechanics refers to the laws of motion developed by Isaac Newton in the 1600s. When the Bohr model of the atom could not predict the energy levels of electrons in atoms with more than one electron, it seemed a new approach to explaining the behavior of electrons was necessary. Developed in the early 1900s, this new approach was based on the work of many scientists. The new approach came to be known as quantum mechanics (also called wave mechanics). **Quantum mechanics** is the branch of physics that deals with the behavior of matter at the atomic and subatomic level.

Properties of Waves

The controversy over the nature of light in the 1600s was partially due to the fact that different experiments with light gave different indications about the nature of light. Energy waveforms, such as water waves or sound waves,

were found to exhibit certain characteristics, including **diffraction** (the bending of waves around corners) and **interference** (the adding or subtracting of energies when waves overlap).



In the image above, the sketch on the left shows a series of ocean wave crests (the long straight blue lines) striking a sea wall. The sea wall has a gap between the cement barriers, which allows the water waves to pass through. The energy of the water waves passes through the gap and essentially bends around the corners so that the continuing waves are now circular and spread out on either side of the gap. The photograph on the right shows a real example of water waves diffracting through a gap between small rocks. This type of behavior is characteristic of energy waveforms.



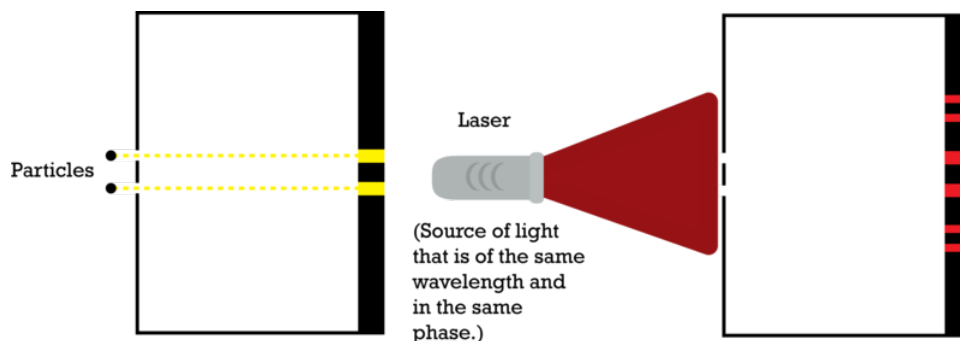
When a body of water has more than one wave, like in the image above, the different waves will overlap and create a new wave pattern. For simplicity, we will consider a case of two water waves with the same amplitude. If the crests of both waves line up, then the new wave will have an amplitude that is twice that of the original waves. The superposition of two crests is represented in the image above as a light area. Similarly, if a trough superimposes over another trough, the new trough will be twice as deep (the darker areas in the image above). In both cases, the amplitude of the new wave is greater than the amplitudes of the individual waves.

Now imagine what would happen if the crest from one water wave is superimposed on a trough from another wave. If both waves are of equal amplitude, the upward pull of the crest and the downward pull of the trough will cancel out. In this case, the water in that area will be flat. The amplitude of this new wave is smaller than the amplitudes of the individual waves.

This process of superimposing waves that occupy the same space is called interference. When the amplitude of the new wave pattern is greater than the amplitudes of the individual waves, it is called constructive interference. When the amplitude of the new wave pattern is smaller than the amplitudes of the individual waves, it is called destructive interference. Interference behavior occurs with all energy waveforms.

Light as a Wave

Light also undergoes diffraction and interference. These characteristics of light can be demonstrated with what is called a double-slit experiment. A box is sealed on all sides so that no light can enter. On one side of the box, two very thin slits are cut. A light source placed in front of the slits will allow light to enter the two slits and shine on the back wall of the box.



If light behaved like particles, the light would go straight from the slits to the back of the box and appear on the back wall as two bright spots (see the left side of the image above). If light behaved like waves, the waves would enter the slits and diffract. On the back wall, an interference pattern would appear with bright spots showing areas of constructive interference and dark spots showing areas of destructive interference (see the right side of the image above). When this double-slit experiment was conducted, researchers saw an interference pattern instead of two bright spots, providing reasonably conclusive evidence that light behaves like a wave.

Light as a Particle

Although the results of the double-slit experiment strongly suggested that light is a wave, a German physicist named Max Planck found experimental results that suggested light behaved more like a particle when he was studying black body radiation. A black body is a theoretical object that absorbs all light that falls on it. It reflects no radiation and appears perfectly black. **Black body radiation** is the energy that would be emitted from an ideal black body. In the year 1900, Planck published a paper on the electromagnetic radiation emitted from a black object that had been heated. In trying to explain the black body radiation, Planck determined that the experimental results could not be explained with the wave form of light. Instead, Planck described the radiation emission as discrete bundles of energy, which he called quanta. A **quantum** (singular form of quanta) is a small unit into which certain forms of energy are divided. These “discrete bundles of energy” once again raised the question of whether light was a wave or a particle – a question once thought settled by Maxwell’s work. Planck’s work also pointed out that the energy of a quantum of light was related only to its frequency. Planck’s equation for calculating the energy of a bundle of light is $E = hf$, where E is the energy of the photon in joules (J), f is the frequency in hertz (s^{-1}), and h is Planck’s constant, $6.63 \times 10^{-34} \text{ J} \cdot \text{s}$. (A **photon** is a particle of light. The word quantum is used for energy in any form; when the type of energy under discussion is light, the words quantum and photon become interchangeable.)

Example:

What is the frequency of a photon of light whose energy is 3.00×10^{-19} joules?

$$f = \frac{E}{h} = \frac{3.00 \times 10^{-19} \text{ J}}{6.63 \times 10^{-34} \text{ J} \cdot \text{s}} = 4.52 \times 10^{14} \text{ Hz}$$

Example:

What is the energy of a photon whose frequency is $2.00 \times 10^{15} \text{ s}^{-1}$?

$$E = hf = (6.63 \times 10^{-34} \text{ J} \cdot \text{s}) \cdot (2.00 \times 10^{15} \text{ s}^{-1}) = 1.33 \times 10^{-18} \text{ J}$$

Planck's work became the basis for quantum theory. **Quantum theory** is the theory that energy can only exist in discrete amounts (quanta). For example, we assume that we can cause an automobile to travel any speed we choose. Quantum theory says this is not true. The problem involved in demonstrating this theory is that the scale of a quantum of energy is much smaller than the objects we normally deal with. Imagine having a large delivery truck sitting on a scale. If we throw one more molecule onto the truck, can we expect to see the weight change on the scale? We cannot, because we lack instruments that can detect such a small change. Even if we added a thousand molecules to the truck, we still would not see a difference in the truck's weight.

For the same reason, we cannot tell that an automobile's speed is quantized (in discrete amounts). The addition of one quantum of kinetic energy to an automobile might change its velocity from 30.111111111 miles per hour to 30.111111122 miles per hour. Therefore, according to quantum theory, a speed of 30.111111117 mph is *not* possible. (Note that these numbers are used for illustrative purposes only.) We are not able to detect this change because we can't measure speeds that finely. To test this theory, we must look at objects that are very tiny in order to detect a change in one quantum.

One place where we can measure quantum-sized energy changes is in the internal vibration of molecules, or the stretching and contracting of bond lengths. When the internal vibration of molecules is measured in the laboratory, it is found that the vibration motion is stair-stepped. A particular molecule may be found vibrating at 3 cycles per second 6 cycles per second or 9 cycles per second, but those molecules are *never* found vibrating at 1, 2, 4, 5, 7, or 8 cycles per second. (Again, these numbers are used for illustrative purposes only.) The fact that only certain vibration levels are available to molecules is strong support for the quantum theory.

Quantum theory can also be used to explain the result of this next experiment on light called the photoelectric effect. The **photoelectric effect** is a phenomenon in which electrons are emitted from the surface of a material after the absorption of energy. This experiment involves having light strike a metal surface with enough force to knock electrons off the metal surface. The results of the photoelectric effect indicated that if the experimenter used low frequency light, such as red, no electrons were knocked off the metal. No matter how many light waves were used and no matter how long the light was shined on the metal, red light could not knock off any electrons. If a higher frequency light was used, such as blue light, then many electrons were knocked off the metal. Albert Einstein used Planck's quantum theory to provide the explanation for the photoelectric effect. A certain amount of energy was necessary for electrons to be knocked off a metal surface. If light were quantized, then only particles of higher frequency light (and therefore higher energy) would have enough energy to remove an electron. Light particles of lower frequency (and therefore lower energy) could never remove any electrons, regardless of how many of them were used.

As a historical side note, many people may think that Einstein won the Nobel Prize for his theory of relativity, but in fact Einstein's only Nobel Prize was for his explanation of the photoelectric effect.

Wave-Particle Duality

At this point, scientists had some experimental evidence (diffraction and interference) that indicated light was a wave and other experimental evidence (black body radiation and the photoelectric effect) that indicated light was a particle. The solution to this problem was to develop a concept known as the **wave-particle duality** of light. The point of this concept is that light travels as a wave and interacts with matter like a particle. Thus when light is traveling through space, air, or other media, we speak of its wavelength and frequency, and when the light interacts with matter, we switch to the characteristics of a particle (quantum).

Lesson Summary

- The work of many scientists led to an understanding of the wave-particle duality of light.
- Light has properties of waves and particles.
- Some characteristics of energy waveforms are that they will undergo diffraction and interference.
- The energy and frequency of a light photon are related by the equation $E = hf$.

Further Reading / Supplemental Links

This website describes the double-slit experience and provides a simulation of the double-slit experiment.

- <http://www.colorado.edu/physics/2000/schroedinger/two-slit2.html>

Review Questions

1. Name a phenomenon that supports the concept that light is a wave.
2. Name a phenomenon that supports the concept that light is a packet of energy.
3. Calculate the energy in joules of a photon whose frequency is 7.55×10^{14} Hz.

1.2 Characteristics of Matter

Lesson Objectives

The student will:

- describe a standing wave.
- state the Heisenberg uncertainty principle.

Vocabulary

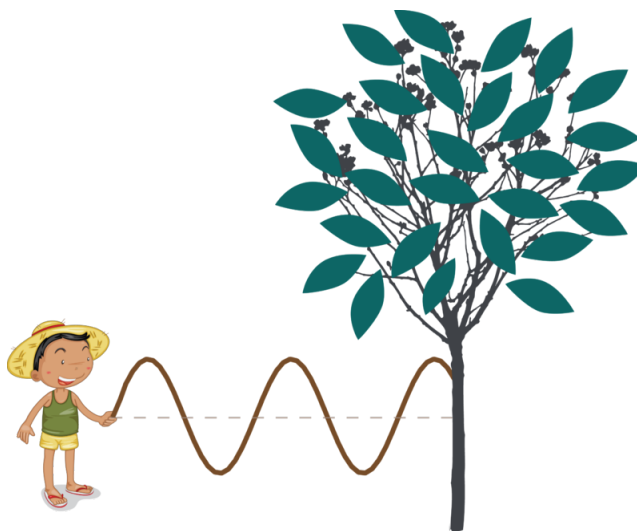
- Heisenberg uncertainty principle

Wave Character of Particles

In 1924, the Frenchman Louis de Broglie, a physics graduate student at the time, suggested that if waves can have particle-like properties as hypothesized by Planck, then perhaps particles can have some wave-like properties. This concept received some experimental support in 1937 when investigators demonstrated that electrons could produce diffraction patterns. (All objects, including baseballs and automobiles could be considered to have wave-like properties, but this concept is only measurable when dealing with extremely small particles like electrons.) De Broglie's "matter waves" would become very useful in attempts to describe the behavior of electrons inside atoms.

Standing Waves

In the chapter "The Bohr Model of the Atom," we considered a rope wave that was created by tying one end of the rope to a tree and by jerking the other end up and down. When a wave travels down a rope and encounters an immovable boundary, the wave reflects off the boundary and travels back up the rope. This causes interference between the wave traveling toward the tree and the reflected wave traveling back toward the person. If the person moving the rope up and down adjusts the rhythm just right, the crests and troughs of the wave moving toward the tree will coincide exactly with the crests and troughs of the reflected wave. When this occurs, the apparent horizontal motion of the crests and troughs along the rope will cease. This is called a standing wave. In such a case, the crests and troughs will remain in the exact same place, while the nodes between the crests and troughs do not appear to move at all.



In the standing wave shown above, the positions of the crests and troughs remain in the same positions. The crests and troughs will only appear to exchange places above and below the rope. The places where the rope does not cross the center axis line are called nodes (positions of zero displacement). These nodal positions do not change and appear to be frozen in place. By combining the concept of a standing wave along with de Broglie's matter waves, it became possible to describe an electron in an electron cloud as either a particle or a standing wave.

The Heisenberg Uncertainty Principle

In all previous attempts to describe the electron's behavior inside an atom, including in the Bohr model, scientists tried to describe the path the electron would follow around the nucleus. The theorists wanted to describe where the electron was located and how it would move from that position to its next position.

In 1927, a German physicist named Werner Heisenberg, a German physicist stated what is now known as the **Heisenberg uncertainty principle**. This principle states that it is impossible to know both the precise location and the precise velocity of an electron at the same time. The reason that we can't determine both is because the act of determining the location changes the velocity. In the process of making a measurement, we have actually changed the measurement.

This problem is present in all laboratory work, but it is usually negligible. Consider the act of measuring the temperature of hot water in a beaker. When you insert the thermometer into the water, the water transfers heat to the thermometer until the thermometer is at the same temperature as the water. You can then read the temperature of the water from the thermometer. The temperature of the water, however, is no longer the same as before you inserted the thermometer. The water has cooled by transferring some of its heat to the thermometer. In other words, the act of making the measurement changes the measurement. In this example, the difference is most likely not significant. You can imagine, however, that if the mass of water was very small and the thermometer was very large, the water would have to transfer a greater amount of heat to the thermometer, resulting in a less accurate measurement.

Consider the method that humans use to see objects. We see an object when photons bounce off the object and into our eye or other light-measuring instrument. Recall that photons can have various wavelengths, which correspond to different colors. If only red photons bounce back, we say the object is red. If no photons bounce back, we don't see the object. Suppose for a moment that humans were gigantic stone creatures that use golf balls, instead of photons, to see. In other words, we see objects when the golf balls bounce off them and enter our eyes. We would be able to see large objects like buildings and mountains successfully, because the golf balls would bounce off and reach our eyes. Could we see something small like a butterfly with this technique? The answer is no. A golf ball has a greater mass than a butterfly, so when the golf ball bounces off the butterfly, the motion of the butterfly will be very different

after the collision. We will know the position of the butterfly, but we won't know the motion of the butterfly.

In the case of electrons, the photons we use to see them with are of significant energy compared to electrons and will change the motion of the electrons upon collision. We may be able to detect the position of the electron, but its motion is no longer the same as before the observation. The Heisenberg uncertainty principle tells us we cannot be sure of both the location and the motion of an electron at the same time. As a result, we must give up on the idea of determining the path an electron follows inside an atom.

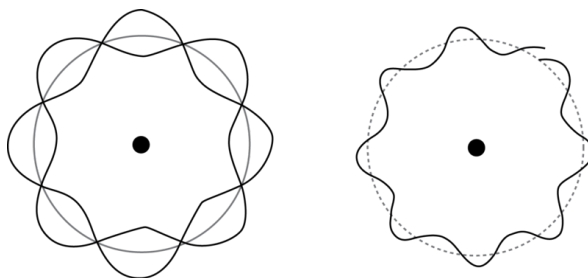
Schrödinger's Equation

The Heisenberg uncertainty principle treated the electron as a particle. In effect, the uncertainty principle stated that the exact motion of an electron in an atom could never be determined, which also meant that the exact structure of the atom could not be determined. Consequently, Erwin Schrödinger, an Austrian physicist, decided to treat the electron as a wave in accordance with de Broglie's matter waves.

Schrödinger, in considering the electron as a wave, developed an equation to describe the electron wave behavior in three dimensions (shown below). Unfortunately, the equation is so complex that it is actually *impossible* to solve exactly for atoms and ions that contain more than one electron. High-speed computers, however, can produce very, very close approximations, and these "solutions" have provided a great deal of information about the possible organization of electrons within an electron cloud.

$$\frac{\overset{\substack{\text{Second derivative} \\ \text{with respect to } x}}{\partial^2 \psi}}{\underset{\substack{\text{Position}}}{\partial x^2}} + \frac{\overset{\substack{\text{Schrodinger} \\ \text{Wave Function}}}{8\pi^2 m}}{h^2} (\overset{\substack{\text{Energy}}}{E} - \overset{\substack{\text{Potential Energy}}}{V}) \psi = 0$$

When we represent electrons inside an atom, quantum mechanics requires that the wave must "fit" inside the atom so that the wave meets itself with no overlap. In other words, the "electron wave" inside the atom must be a standing wave. If the wave is to be arranged in the form of a circle so that it attaches to itself, the waves can only occur if there is a whole number of waves in the circle. Consider the image below.



On the left is an example of a standing wave. For the wave on the right, the two ends of the wave do not quite meet each other, so the wave fails to be a standing wave. There are only certain energies (frequencies) for which the wavelength of the wave will fit exactly to form a standing wave. These energies turn out to be the same as the energy levels predicted by the Bohr model, but now there is a reason why electrons may only occupy these energy

levels. (Recall that one of the problems with the Bohr model was that Bohr had no explanation for why the electrons could only occupy certain energy levels in the electron cloud.) The equations of quantum mechanics tell us about the existence of principal energy levels, the number of energy levels in any atom, and more detailed information about the various energy levels.

Lesson Summary

- The Heisenberg uncertainty principle states that it is impossible to know both the precise location and the precise motion of an electron at the same time.
- Electrons in an electron cloud can be viewed as a standing wave.
- The reason that an electron in an atom may have only certain energy levels is because only certain energies of electrons will form standing waves in the enclosed volume.
- The solutions to Schrödinger's equation provide a great deal of information about the organization of the electrons in the electron cloud.

Further Reading / Supplemental Links

A question and answer session on electrons behaving as waves.

- <http://www.colorado.edu/physics/2000/quantumzone/debroglie.html>

Review Questions

1. Which of the following statements are true?
 - a. According to the Heisenberg uncertainty principle, we will eventually be able to measure both an electron's exact position and its exact momentum at the same time.
 - b. The problem that we have when we try to measure an electron's exact position and its exact momentum at the same time is that our measuring equipment is not good enough.
 - c. According to the Heisenberg uncertainty principle, we cannot know both the exact location and the exact momentum of an automobile at the same time.
 - d. The Heisenberg uncertainty principle applies only to very small objects like protons and electrons.
 - e. The Heisenberg uncertainty principle applies only to large objects like cars and airplanes.
 - f. The Heisenberg Heisenberg uncertainty principle applies to very small objects like protons and electrons and to large objects like cars and airplanes.

1.3 Quantum Numbers, Orbitals, and Probability Patterns

Lesson Objectives

The student will:

- state the relationship between the principal quantum number (n), the number of orbitals, and the maximum number of electrons in a principal energy level.

Vocabulary

- Pauli exclusion principle
- principal quantum number
- quantum number

Introduction

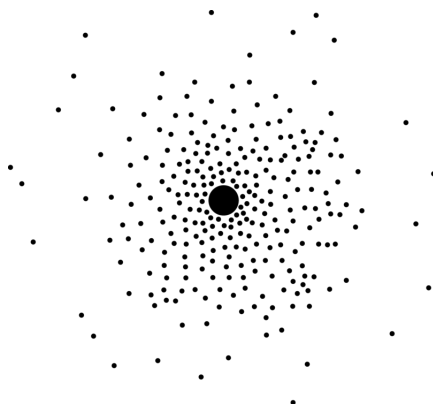
Erwin Schrödinger proposed a wave equation for electron matter waves that was similar to the known equations for other wave motions in nature. This equation describes how a wave associated with an electron varies in space as the electron moves under various forces. Schrödinger worked out the solutions of his equation for the hydrogen atom, and the results agreed perfectly with the known energy levels for hydrogen. Furthermore, the equation could be applied to more complicated atoms. It was found that Schrodinger's equation gave a correct description of an electron's behavior in almost every case. In spite of the overwhelming success of the wave equation in describing electron energies, the very meaning of the waves was vague and unclear.

There are very few scientists who can visualize the behavior of an electron as a standing wave during chemical bonding or chemical reactions. When chemists are asked to describe the behavior of an electron in an electrochemical cell, they do not use the mathematical equations of quantum mechanics, nor do they discuss standing waves. The behavior of electrons in chemical reactions is best understood by considering the electrons to be particles.

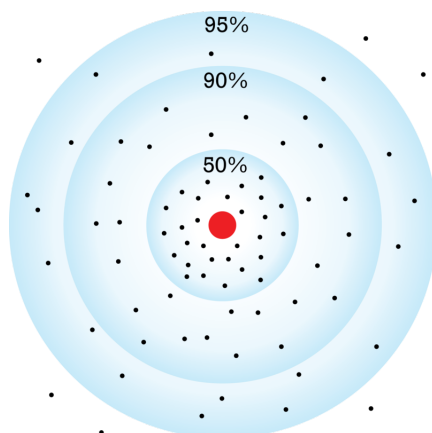
A physicist named Max Born was able to attach some physical significance to the mathematics of quantum mechanics. Born used data from Schrodinger's equation to show the probability of finding the electron (as a particle) at the point in space for which the equation was solved. Born's ideas allowed chemists to visualize the results of Schrodinger's wave equation as probability patterns for electron positions.

Probability Patterns

Suppose we had a camera with such a fast shutter speed that it could capture the position of an electron at any given moment. We could take a thousand pictures of this electron at different times and find it at many different positions in the atom. We could then plot all the electron positions onto one picture, as seen in the sketch below.

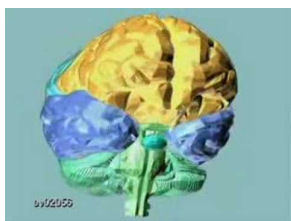


One way of looking at this picture is as an indication of the probability of where you are likely to find the electron in this atom. Keep in mind that this image represents an atom with a single electron. The dots do not represent different electrons; the dots are positions where the single electron can be found at different times. From this image, it is clear that the electron spends more time near the nucleus than it does far away. As you move away from the nucleus, the probability of finding the electron becomes less and less. It is important to note that there is no boundary in this picture. In other words, there is no distance from the nucleus where the probability of finding an electron becomes zero. However, for much of the work we will be doing with atoms, it is convenient (even necessary) to have a boundary for the atom. Most often, chemists arbitrarily draw in a boundary for the atom, choosing some distance from the nucleus beyond which the probability of finding the electron becomes very low. Frequently, the boundary is placed such that 90% of the probability of finding the electron is inside the boundary.



The image above shows boundaries drawn in at 50%, 90%, and 95% probability of finding the electron within the boundary. It is important to remember that the boundary is there for our convenience, and there is no actual boundary on an atom. This probability plot is very simple because it is for the first electron in an atom. As the atoms become more complicated (more energy levels and more electrons), the probability plots also become more complicated.

All of the scientists whose names appear in the "Atom Song" have appeared in our book. Please watch the video: <http://www.youtube.com/watch?v=vUzTQWn-wfE> (3:28).



MEDIA

Click image to the left for more content.

The Principal Quantum Number

Solutions to Schrödinger's equation involve four special numbers called **quantum numbers**. (Three of the numbers come from Schrödinger's equation, and the fourth one comes from an extension of the theory.) These four numbers completely describe the energy of an electron. Each electron has exactly four quantum numbers, and no two electrons have the same four numbers. The statement that no two electrons can have the same four quantum numbers is known as the **Pauli exclusion principle**.

The **principal quantum number** is a positive integer (1, 2, 3, . . . n) that indicates the main energy level of an electron within an atom. According to quantum mechanics, every principal energy level has one or more sub-levels within it. The number of sub-levels in a given energy level is equal to the number assigned to that energy level. That is, principal energy level 1 will have 1 sub-level, principal energy level 2 will have two sub-levels, principal energy level 3 will have three sub-levels, and so on. In any energy level, the maximum number of electrons possible is $2n^2$. Therefore, the maximum number of electrons that can occupy the first energy level is 2 ($2 \cdot 1^2$). For energy level 2, the maximum number of electrons is 8 ($2 \cdot 2^2$), and for the 3rd energy level, the maximum number of electrons is 18 ($2 \cdot 3^2$). **Table 1.1** lists the number of sub-levels and electrons for the first four principal quantum numbers.

TABLE 1.1: Number of Sub-levels and Electrons by Principal Quantum Number

Principal Quantum Number	Number of Sub-Levels	Total Number of Electrons
1	1	2
2	2	8
3	3	18
4	4	32

The largest known atom contains slightly more than 100 electrons. Quantum mechanics sets no limit as to how many energy levels exist, but no more than 7 principal energy levels are needed to describe the electrons of all known atoms. Each energy level can have as many sub-levels as the principal quantum number, as discussed above, and each sub-level is identified by a letter. Beginning with the lowest energy sub-level, the sub-levels are identified by the letters *s*, *p*, *d*, *f*, *g*, *h*, *i*, and so on. Every energy level will have an *s* sub-level, but only energy levels 2 and above will have *p* sub-levels. Similarly, *d* sub-levels occur in energy level 3 and above, and *f* sub-levels occur in energy level 4 and above. Energy level 5 could have a fifth sub-energy level named *g*, but all the known atoms can have their electrons described without ever using the *g* sub-level. Therefore, we often say there are only four sub-energy levels, although theoretically there can be more than four sub-levels. The principal energy levels and sub-levels are shown in the following diagram. The principal energy levels and sub-levels that we use to describe electrons are in red.

Energy Levels	Sub-Levels							
1	s							
2	s	p						
3	s	p	d					
4	s	p	d	f				
5	s	p	d	f	g			
6	s	p	d	f	g	h		
7	s	p	d	f	g	h	i	
8	s	p	d	f	g	h	i	j

Orbitals

Quantum mechanics also tells us how many orbitals are in each sub-level. In Bohr's model, an orbit was a circular path that the electron followed around the nucleus. In quantum mechanics, an orbital is defined as an area in the electron cloud where the probability of finding the electron is high. The number of orbitals in an energy level is equal to the square of the principal quantum number. Hence, energy level 1 will have 1 orbital (1^2), energy level 2 will have 4 orbitals (2^2), energy level 3 will have 9 orbitals (3^2), and energy level 4 will have 16 orbitals (4^2).

The *s* sub-level has only one orbital. Each of the *p* sub-levels has three orbitals. The *d* sub-levels have five orbitals, and the *f* sub-levels have seven orbitals. If we wished to assign the number of orbitals to the unused sub-levels, *g* would have nine orbitals and *h* would have eleven. You might note that the number of orbitals in the sub-levels increases by odd numbers (1, 3, 5, 7, 9, 11, . . .).

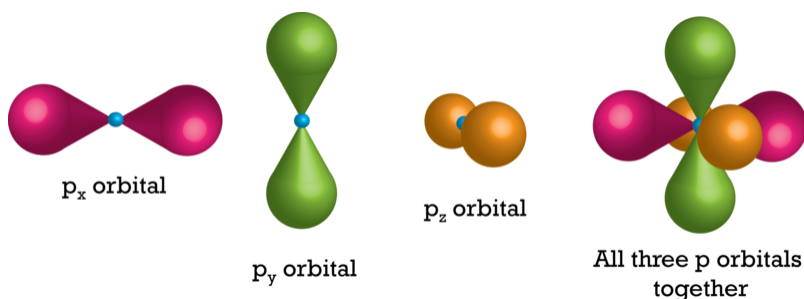
As a result, the single orbital in energy level 1 is the *s* orbital. The four orbitals in energy level 2 are a single *2s* orbital and three *2p* orbitals. The nine orbitals in energy level 3 are a single *3s* orbital, three *3p* orbitals, and five *3d* orbitals. The sixteen orbitals in energy level 4 are a the single *4s* orbital, three *4p* orbitals, five *4d* orbitals, and seven *4f* orbitals.

Principal Energy Level (<i>n</i>)	Number of Orbitals Present				Total Number of Orbitals (n^2)	Maximum Number of Electrons ($2n^2$)
	s	p	d	f		
1	1	-	-	-	1	2
2	1	3	-	-	4	8
3	1	3	5	-	9	18
4	1	3	5	7	16	32

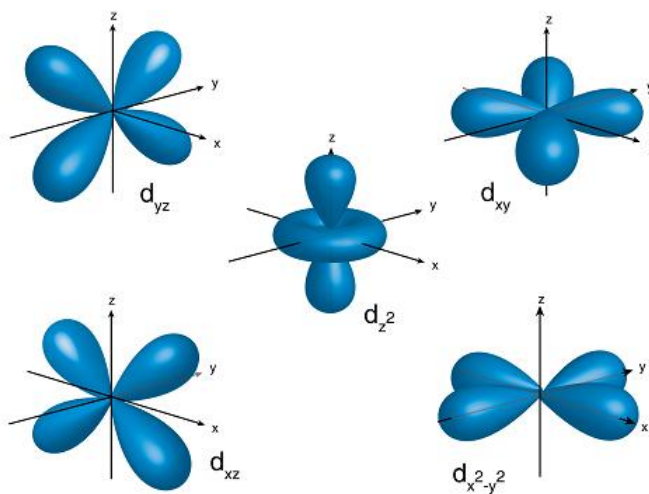
The chart above shows the relationship between *n* (the principal quantum number), the number of orbitals, and the maximum number of electrons in a principal energy level. Theoretically, the number of orbitals and number of electrons continue to increase for higher values of *n*. However, no atom actually has more than 32 electron in any of its principal levels.

Each orbital will also have a probability pattern that is determined by interpreting Schrödinger's equation. Earlier, we showed that the probability pattern for an atom with a single electron is a circle. The illustration, however, is 2-dimensional. The real 3-dimensional probability pattern for the single orbital in the s sub-level is actually a sphere.

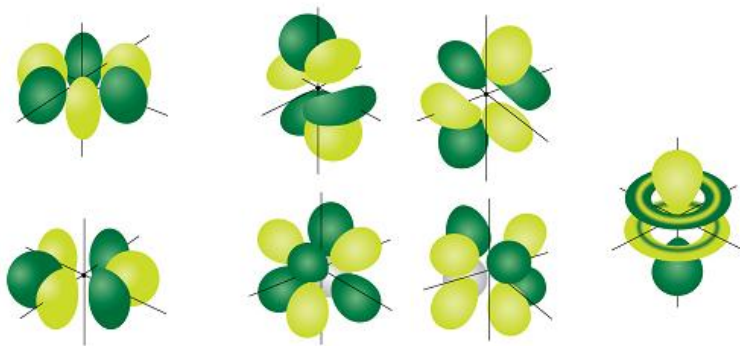
The probability patterns for the three orbitals in the p sub-levels are shown below. The three images on the left show the probability pattern for the three p orbitals in each of the three dimensions. On the far right is an image of all three p orbitals together. These p orbitals are said to be shaped like dumbbells (named after the objects weight lifters use), water wings (named after the floating balloons young children use in the swimming pool), and various other objects.



The probability patterns for the five d orbitals are more complicated and are shown below.

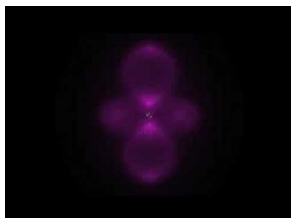


The seven f orbitals shown below are even more complicated.



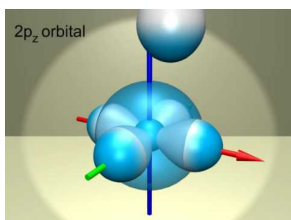
You should keep in mind that no matter how complicated the probability pattern is, each shape represents a single orbital, and the entire probability pattern is the result of the various positions that either one or two electrons can take.

A video discussing the relationship between spectral lines and electron transitions is available at (1j) <http://www.youtube.com/watch?v=fKYso97eJs4> (3:49).

**MEDIA**

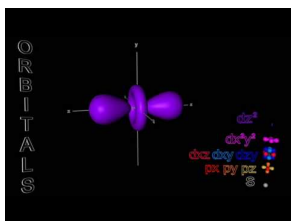
Click image to the left for more content.

A short animation of s and p orbitals is available on youtube.com at <http://www.youtube.com/watch?v=VfBcfYR1VQo> (1:20).

**MEDIA**

Click image to the left for more content.

Another example of s, p and d electron orbitals is available also on youtube.com at <http://www.youtube.com/watch?v=K-jNgq16jEY> (1:37).

**MEDIA**

Click image to the left for more content.

Lesson Summary

- Solutions to Schrodinger's equation involve four special numbers called quantum numbers, which completely describe the energy of an electron.
- Each electron has exactly four quantum numbers.
- According to the Pauli Exclusion Principle, no two electrons have the same four quantum numbers.
- The major energy levels are numbered by positive integers (1, 2, 3, . . . , n), and this number is called the principal quantum number
- Quantum mechanics also tells us how many orbitals are in each sub-level.
- In quantum mechanics, an orbital is defined as an area in the electron cloud where the probability of finding the electron is high.

Further Reading / Supplemental Links

The following is a video on the quantum mechanical model of the atom.

- http://www.youtube.com/watch?v=IsA_oIXdF_8&feature=related

This video is a ChemStudy film called “Hydrogen Atom and Quantum Mechanics.” The film is somewhat dated but the information is accurate. The video also contains some data supporting quantum theory.

- <http://www.youtube.com/watch?v=80ZPe80fM9U>

Review Questions

1. How many sub-levels may be present in principal energy level 3 ($n = 3$)?
2. How many sub-levels may be present in principal energy level 6 ($n = 6$)?
3. Describe the difference in the definitions of a Bohr orbital and a quantum mechanics orbital.
4. What is the maximum total number of electrons that can be present in an atom having three principal energy levels?

In the first image of this chapter, the photograph showing water waves diffracting through a gap between small rocks is from <http://www.flickr.com/photos/framesofmind/554402976/> (CC-BY-SA).

All images, unless otherwise stated, are created by the CK-12 Foundation and are under the Creative Commons license CC-BY-NC-SA.